A Frame Design for MIMO UW based Systems: Overhead Analysis & Channel Estimation

Shahab Ehsanfar*, Marwa Chafii**, *IEEE Member*, Gerhard Fettweis*, *IEEE Fellow*

*Vodafone Chair Mobile Communications Systems, Technische Universität Dresden, Germany
**ETIS UMR 8051, Université Paris-Seine, Université de Cergy-Pontoise, ENSEA, CNRS, France
{*shahab.ehsanfar, fettweis*}@*ifn.et.tu-dresden.de*, *marwa.chafii@ensea.fr*

***Abstract*—In this paper, we extend the conventional single-input-single-output unique word (UW) based frame designs to MIMO applications with spatial multiplexing. Considering a doubly-dispersive channel where the channel is varying both in time and frequency, we analyze the overhead requirements of such systems for synchronization and channel estimation. Through numerical and simulation results, we show that the proposed MIMO UW based frame design, achieves a higher transmission efficiency than the traditional cyclic prefix approach as in orthogonal frequency division multiplexing (CP-OFDM). Derivations of the linear minimum mean squared error channel estimation of MIMO UW sequences show that the channel estimation performance can be highly improved with respect to CP-OFDM systems in doubly-dispersive channels.**

I. Introduction

FLEXIBLE numerology of the physical layer has been introduced in the latest release of 5G NR, while the baseline waveform generation is chosen to be cyclic-prefix based orthogonal frequency division multiplexing (CP-OFDM). Thanks to the narrow subcarrier spacing and low complexity one tap equalization of OFDM, it suits well to time-dispersive channels. Having the CP length longer than the channel delay spread, the wireless channel can be interpreted as a circulant channel at the receiver side, assuming that the channel remains almost stationary during the OFDM symbol. On the other hand, for the upcoming 5G and beyond use-case scenarios (e.g. vehicle-to-everything (V2X), unmanned aerial vehicles, etc.), it is envisioned that the users may experience high mobility conditions. While the frame structures of the LTE and 5G NR are being designed for long coherence times, the orthogonality principles of OFDM remain valid for low mobility use-cases. In reality however, the users' mobility does not always remain at a constant pace. In this case, the coherence time experienced by the transceiver starts to vary, and if it becomes too short (due to high mobility), OFDM subcarriers lose their orthogonality and the channel circularity assumptions do not hold anymore. In addition, since the CP part of the CP-OFDM is a random signal that changes from symbol to symbol, it provides very limited advantages from synchronization and channel estimation perspectives.

Replacing the CP by a deterministic and a known sequence—so called unique-word (UW)—was initially proposed by [1] and [2] for single carrier (SC) systems. Thanks to the repetition of the UW sequences in all transmission blocks, and their full knowledge at the receiver side, the receiver can take advantage of per-block synchronization and channel estimation while it also gains some equalization performances. To this end, further number of studies have been proposed for UW based single and multi-carrier systems. For instance, [3] and [4] proposed different approaches of channel estimation in single-input single-output (SISO) UW-SC systems. Their performance evaluation however, was applied to low mobility and additive white Gaussian noise (AWGN) channel conditions, respectively. Nevertheless, all the above related works were focusing on SISO systems and their approaches are not straightforwardly applicable to multiple-input multiple-output (MIMO) multi-carrier systems, because in MIMO, the UW sequences transmitted from each antenna not only must cover the entire band where payloads are present, but also must it be orthogonal modulatable sequences (i.e. their periodic auto-correlation function must be a Kronecker's Dirac delta function), whereas, the cross-correlation between the sequences transmitted from different antennas must be constant, accommodating a high quality MIMO synchronization [5] and channel estimation.

In this paper, by taking into account the UW frame design of [1] as a benchmark, we extend the frame structure to MIMO applications. Considering doubly-dispersive channel conditions where the wireless channel is both frequency-selective and time-variant, we analyze the overhead requirements of the UW based system from synchronization and channel estimation perspectives and we compare the overhead requirements with a pilot-aided CP-OFDM system design. Given the time varying channel conditions and via extensive simulations, we show that the proposed MIMO frame structure, achieves a significantly higher performance than the conventional pilot-aided channel estimation techniques in CP-OFDM systems.

The rest of this paper is organized as follows: In Sec. II, we recap the state-of-the-art approaches from a UW based system design to MIMO CP-OFDM frame design in high mobility channel conditions. In Sec. III, which is two-fold, we initially extend the SISO frame structure to MIMO systems and we analyze its corresponding overhead and transmission efficiency. In the second part of Sec. III, we derive the corresponding LMMSE based channel estimation of the MIMO UW sequences. Next, we evaluate the performance of the proposed

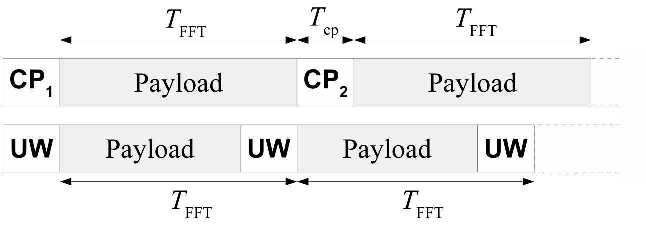

Fig. 1. UW vs. CP design according to [1]

approaches numerically and via simulations in Sec. IV. Finally, conclusions are drawn in Sec. V.

II. State of the Art

A. Unique Words in SISO systems

In a UW based system [1], instead of a cyclic prefix (CP), a deterministic sequence is added as a prefix and also as a suffix around the transmission block (e.g. see Fig. 1). Therefore, the symbol duration reduces from $N + N_{\rm cp}$ to N samples. Here, $N_{\rm cp}$ is the number of time-domain samples of the CP, and N is the data fast Fourier transform (FFT) size. Similar to CP based systems, if the UW length N_u is longer than the channel delay spread with length L, and also the channel remains constant during the block duration of N samples, the receiver can interpret a circular channel transfer function and perform FFT based channel estimation and equalization approaches. Moreover, since UW sequences are deterministic and known, per-block synchronization can be achieved.

However, from a channel estimation perspective, if the UW length $N_{\rm u}$ is set to L samples of the channel delay spread length, the UW observations would always be interference limited, because taking the FFT of the frame over N samples of $T_{\rm FFT}$ duration, payload becomes overlapped with UW sequences in frequency domain. In [3], authors suggested that the UW length must be at least twice of the channel length (i.e. [3] $N_u = 2L$) to achieve an interfrerence-free observation at the second half of the UW. Although, in order to use the energy of the UW from the first half, they suggested an iterative interference cancellation approach. Unfortunately, the performance of the approach in [3] was not compared to an equivalent CP based system and moreover, it is also only applicable to low mobility channel conditions.

In the following, describing a typical MIMO scenario, we provide an example on how a conventional CP-OFDM design can handle a doubly-dispersive channel and in Sec. III, we show that MIMO UW can handle such channel conditions in a more efficient fashion.

B. MIMO frames based on UW

Consider a centralized MIMO system with $\mathsf{n_T} \times \mathsf{n_R}$ antennas. All the Tx. antennas use a shared oscillator clock source, therefore, the transmit frames are aligned according to Fig. 2. The transmission is initiated by sending a double UW sequence in form of a preamble, which assures a primary time-frequency synchronization and channel estimation. Afterwards blocks of *Payload-UW* (with length $N = N_d + N_u$) are being transmitted at a baudrate F_b. Thanks to the repeated UW sequences at the end of the blocks, the channel transfer function can be interpreted as a circular filter. Nevertheless, the circular channel assumption holds, if the *Payload-UW* block experiences a block-fading scenario, in which the block length of N samples with duration N/F_s—where, F_s denotes the sampling rate—is much smaller than the channel coherence time T_c (typically 2% as in LTE). If the user starts to have mobility, the coherence time that the user experiences also starts to vary. In high mobility conditions, the channel coherence time T_c becomes much smaller than the stationary condition $T_{c,0}$, in which, the *Payload-UW* block duration was designed for. In this case, the channel transfer function starts to vary within the block duration and therefore, the circularity of the wireless channel does not hold any longer. If the block fading assumption breaks down, inter-carrier-interference (ICI) would occur and consequently, it limits the channel estimation and equalization performances.

In addition, in order to enable an accurate MIMO synchronization [5], as well as MIMO channel estimation [6], the UW sequences transmitted from different Tx antennas must be orthogonal sequences with a property such that their auto-correlation functions to be Kronecker's Delta function $\delta[n]$, and their mutual cross-correlation to be a constant value leading to minimized maximum absolute value [7].

C. Illustrative Scenario

For clarity, consider an extreme scenario of a spatial multiplexing 2×2 MIMO system, in which, 100 complex data[1] (i.e. 50 per Tx. antenna) must be transmitted by 18 subcarriers over a frequency selective—with $L = 3$ taps—and fast fading[2] channel. In [8], it is recommended that for a CP-OFDM system in a time-variant channel, it is necessary to employ comb-type pilots pattern, i.e. pilot insertion into every OFDM symbol. In this case $\mathsf{n_T} L = 6$ channel taps must be estimated and therefore, 6 pilot subcarriers must be inserted into every OFDM symbol. From a total number of 18 subcarriers, each OFDM symbol can carry 12 data subcarriers. The frame design for such an OFDM system is depicted in the upper part of Fig. 3. As can be seen in Fig. 3, five CP-OFDM symbols would be necessary to transmit the 100 complex values.

III. MIMO UW Frame Design

Consider again the example scenario of Sec. II-C, in which, the knowledge of UW sequences shall be exploited for channel estimation (and also synchronization), and therefore, no pilot insertion is necessary. In this case, we design the length of the UW sequence to be $N_{\rm u} \geq (\mathsf{n_T} + 1)L$, such that the first L samples shall be ignored (due to the inter-block-interference (IBI) from the previous block), and the rest of $(N_{\rm u} - L) \geq \mathsf{n_T} L$ samples would be used to estimate $\mathsf{n_T} L$ channel parameters. We also note that, the payload size does not necessarily need

[1]Complex data is referred to any complex value from a modulation alphabet, e.g. 2^μ-QAM. For instance, one may assume 100 bits via 1/2 code-rate QPSK.

[2]The channel gets outdated from symbol-to-symbol.

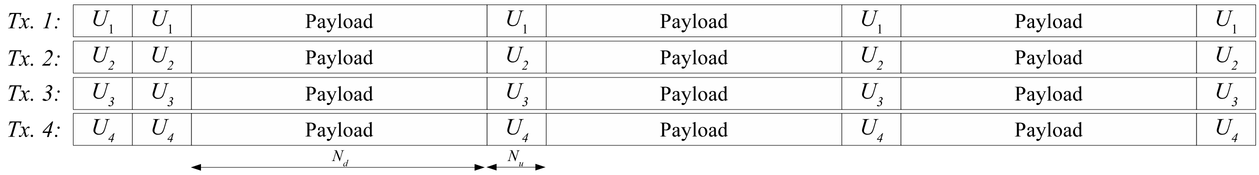

Fig. 2. Example of the frame structure for 4 Tx antennas

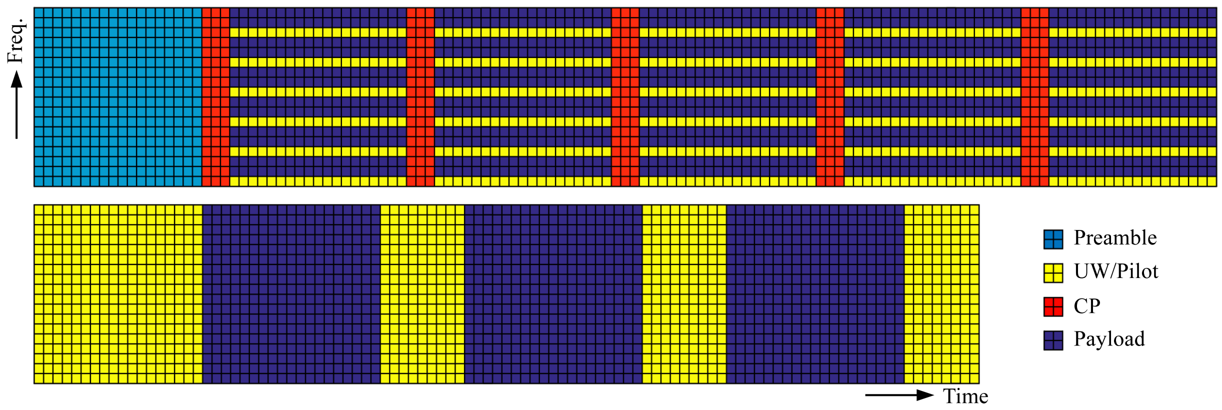

Fig. 3. Example of the frame structure in time-frequency grid. CP based frame needs $5(K+L) = 105$ time resources to transmit 50 complex data, whereas, UW based frame transmits them over $3(K + (\mathsf{n}_\mathsf{T}+1)L) = 81$ time resources.

to be reduced to fit the *Payload-UW* block within the FFT size, and the equalization can be handled analogously as in [9]. In addition, at the beginning of a transmission, we consider a double UW sequence for primary synchronization, which is equivalent to the preamble of the CP-OFDM system. For the rest of the blocks, we consider only a single UW sequence. Note that here, if we use double UW sequence for all blocks, the overhead becomes significant, and if we use double half-length UW sequences, the signal-to-noise ratio (SNR) for synchronization becomes half. The frame design of such a UW based system is depicted in lower part of Fig. 3. There, we can see that despite the overhead could be still large (i.e. 33% without preamble), only three *Payload-UW* blocks would be necessary for transmission of 100 complex data, and thus the time resource of one OFDM symbol is being saved. Nevertheless, the overhead of such frame design is further discussed in the following.

A. CP vs. UW Overhead analysis

For primary synchronization, a preamble consisting of two repetitive deterministic sequences shall be used for both CP-OFDM and UW-OFDM. The CP-OFDM employs a preamble for B_{CP} number of OFDM blocks while in UW-OFDM the preamble is used for $B_{\mathrm{UW}} \geq B_{\mathrm{CP}}$. Thanks to the unique word sequences after each payload block, per-block-synchronization allows B_{UW} to be much larger than B_{CP}.

Summing up the overhead requirements that were discussed in Sec. II-C and beginning of this section, the number of overhead samples for CP-OFDM and UW-OFDM follow:

$$\xi_{\mathrm{CP}} = \underbrace{N_p}_{\text{Preamble}} + B_{\mathrm{CP}}(\underbrace{L}_{\text{CP}} + \underbrace{\mathsf{n}_\mathsf{T} L}_{\text{Pilots}}), \tag{1}$$

$$\xi_{\mathrm{UW}} = \underbrace{N_p}_{\text{Preamble}} + B_{\mathrm{UW}} \underbrace{(\mathsf{n}_\mathsf{T}+1)L}_{\text{UW length}}, \tag{2}$$

respectively. Here, N_p is the preamble size, and for a UW based system, we set it to $N_p = 2N_{\mathrm{u}}$ for two repeated UW sequences. Comparing (1) and (2), one may notice that if the number of transmission blocks B_{CP} and B_{UW} are equal, the number of overhead samples for CP-OFDM and UW-OFDM becomes equal too, i.e. $\xi_{\mathrm{CP}} = \xi_{\mathrm{UW}}$ for $B_{\mathrm{CP}} = B_{\mathrm{UW}}$.

On the other hand, the total resource sizes for both systems follow:

$$\mathcal{S}_{\mathrm{CP}} = N_p + B_{\mathrm{CP}}(L + N_d), \tag{3}$$

$$\mathcal{S}_{\mathrm{UW}} = N_p + B_{\mathrm{UW}}\big((\mathsf{n}_\mathsf{T}+1)L + N_d\big), \tag{4}$$

where, N_d is the payload length.

Clearly, the resource size $\mathcal{S}_{\mathrm{UW}}$ for UW-OFDM is always larger than $\mathcal{S}_{\mathrm{CP}}$ for CP-OFDM, even if the number of blocks B_{UW} and B_{CP} are equal.

Consequently, the transmission efficiency of the two systems is given by

$$\eta_{\mathrm{CP}} = 1 - \frac{\xi_{\mathrm{CP}}}{\mathcal{S}_{\mathrm{CP}}} \tag{5}$$

$$\eta_{\mathrm{UW}} = 1 - \frac{\xi_{\mathrm{UW}}}{\mathcal{S}_{\mathrm{UW}}}. \tag{6}$$

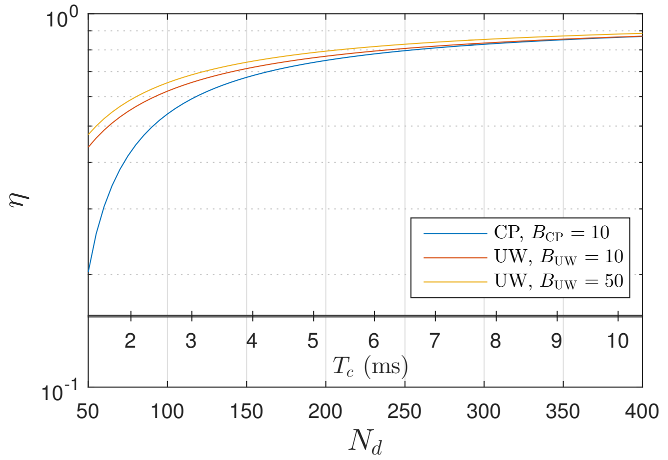

Fig. 4. Transmission Efficiency vs. data block size and channel coherence time.

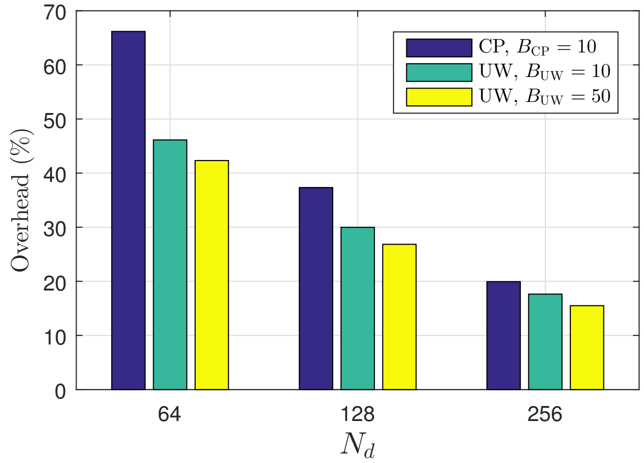

Fig. 5. Overhead comparisons for radix-2 FFT sizes.

Note that, the above transmission efficiency is equivalent to a normalized spectral efficiency per antenna and per modulation and coding scheme (MCS).

B. Channel Estimation

Given the knowledge of the UW sequences at the receiver side, one can estimate the MIMO channel impulse responses via linear minimum mean square error (LMMSE) estimation [10]. For the UW sequences, we choose *orthogonal polyphase sequences* proposed by [7] which are suitable for both MIMO synchronization [5] and channel estimation units. Thus, the UW sequence $x_{\mathrm{u},i_{\mathsf{T}}}[n]$ from Tx antenna i_{T} is given by

$$x_{\mathrm{u},i_{\mathsf{T}}}[n] = \exp\left(\frac{j2\pi\ i_{\mathsf{T}}\ n_0 n_1}{\sqrt{N_{\mathrm{u}}}}\right), \tag{7}$$

where, $n = n_0\sqrt{N_{\mathrm{u}}} + n_1$, for $n_0 \in \mathcal{S}$, $n_1 \in \mathcal{S}$, $\mathcal{S} = \{0, \cdots, \sqrt{N_{\mathrm{u}}} - 1\}$. Collecting the samples of $x_{\mathrm{u},i_{\mathsf{T}}}[n]$ in form of a vector notation $\vec{x}_{\mathrm{u},i_{\mathsf{T}}}$, N_{u} is the length of $\vec{x}_{\mathrm{u},i_{\mathsf{T}}}$ and the square root $\sqrt{N_{\mathrm{u}}}$ must be a prime number [7]. We note that, the periodic auto-correlation function of $\vec{x}_{\mathrm{u},i_{\mathsf{T}}}$ is a Kronecker's Delta function $\delta[n]$, while the periodic cross-correlation of $\vec{x}_{\mathrm{u},i_{\mathsf{T}}}$ and $\vec{x}_{\mathrm{u},i'_{\mathsf{T}}\neq i_{\mathsf{T}}}$ is a constant value equal to $1/\sqrt{N_{\mathrm{u}}}$ [7].

Assuming a perfectly time and frequency synchronized received signal at the last $N_u - L$ samples of the UW slot at antenna i_{R}, we have

$$\vec{y}_{p,i_{\mathsf{R}}} = \boldsymbol{X}_p \vec{h}_{i_{\mathsf{R}}} + \vec{w}_{i_{\mathsf{R}}}, \tag{8}$$

where $\boldsymbol{X}_p = [\boldsymbol{X}_{p,1}, \cdots, \boldsymbol{X}_{p,\mathsf{n_T}}] \in \mathbb{C}^{(N_u-L)\times \mathsf{n_T} L}$ is the observation matrix with $\boldsymbol{X}_{p,i_{\mathsf{T}}} \in \mathbb{C}^{(N_{\mathrm{u}}-L)\times L}$ being the last $(N_{\mathrm{u}} - L)$ rows and first L columns of $\boldsymbol{X}_{u,i_{\mathsf{T}}}$. Here, $\boldsymbol{X}_{u,i_{\mathsf{T}}}$ is a lower triangular Toeplitz matrix with UW sequence $\vec{x}_{\mathrm{u},i_{\mathsf{T}}}$ on its first column. $\vec{h}_{i_{\mathsf{R}}} = [\vec{h}^T_{i_{\mathsf{T}},i_{\mathsf{R}}}]^T_{i_{\mathsf{T}}=1:\mathsf{n_T}} \in \mathbb{C}^{\mathsf{n_T} L}$ denotes the parameters vector of $\mathsf{n_T}$ channel impulse responses of length L. $\vec{w}_{i_{\mathsf{R}}} \in \mathbb{C}^{N_{\mathrm{u}}-L}$ denotes the AWGN process of variance σ_w^2. The LMMSE estimation of $\vec{h}_{i_{\mathsf{R}}}$ follows [10]

$$\hat{\vec{h}}_{i_{\mathsf{R}}} = (\sigma_w^2 \boldsymbol{\Sigma}_{hh}^{-1} + \boldsymbol{X}_p^H \boldsymbol{X}_p)^{-1} \boldsymbol{X}_p^H \vec{y}_{p,i_{\mathsf{R}}}, \tag{9}$$

$$\widehat{\boldsymbol{\Sigma}}_{hh} = \boldsymbol{\Sigma}_{hh} - (\sigma_w^2 \boldsymbol{\Sigma}_{hh}^{-1} + \boldsymbol{X}_p^H \boldsymbol{X}_p)^{-1} \boldsymbol{X}_p^H \boldsymbol{X}_p \boldsymbol{\Sigma}_{hh}, \tag{10}$$

where $\boldsymbol{\Sigma}_{hh} = \mathbb{E}[\vec{h}_{i_{\mathsf{R}}} \vec{h}_{i_{\mathsf{R}}}^H]$ denotes the time-domain channel auto-correlation matrix. Once, an estimate $\hat{\vec{h}}_{i_{\mathsf{R}}}$ for each of both UW sequences over the *UW-Payload-UW* block is obtained, the two estimations are being averaged and fed to the equalization unit.

An advantage of the above LMMSE estimation is that the observation signal $(x_{\mathrm{u},i_{\mathsf{T}}}[n])_{n=L:N_{\mathrm{u}}-1}$ is interference-free from IBI perspective. However, it also has a drawback due to the partial selection of $\vec{x}_{\mathrm{u},i_{\mathsf{T}}}$. The sequence $\vec{x}_{\mathrm{u},i_{\mathsf{T}}}$ has a constant frequency magnitude which is an optimal sequence for channel estimation from an SNR sense. By selecting the last $N_{\mathrm{u}} - L$ samples of $\vec{x}_{\mathrm{u},i_{\mathsf{T}}}$, the signal magnitude in frequency domain decreases at near edge subcarriers. Therefore, in case of a full subcarrier allocation, it would be recommended to use the whole interference-limited $\vec{x}_{\mathrm{u},i_{\mathsf{T}}}$ sequence for channel estimation.

In addition, we should also add that in time-variant scenarios, where the channel varies within the block duration, it is expected that the estimated channel responses have their most similarity to the average of the channel realizations within the block [11], provided that the symbol energy is uniform within its duration.

IV. Numerical Results

A. Transmission Efficiency and Overhead

In this section, we make a numerical comparison of transmission efficiency and overhead of UW based systems vs. CP based systems in a 4×4 MIMO setup. We assume a fast-fading scenario where comb-type pilot pattern is necessary. The sampling frequency is set to $F_s = 1.92$MHz and power-delay-profile (PDP) is assumed to have $L = 9$ taps, which

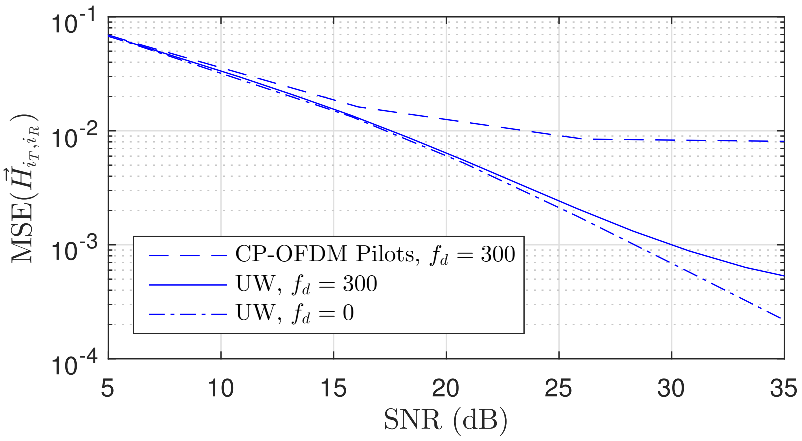

Fig. 6. Channel Estimation Performance.

corresponds to 4.7μs maximum delay spread. The UW length is set to $N_u = 49$ samples and for the sake of primary synchronization a double length UW is used as preamble. In the CP based system we also consider a preamble length of $N_p = 2N_u$ to be used for synchronization and afterwards, $B_{\mathrm{OFDM}} = 10$ blocks are continuously being transmitted. The CP length is set to $N_{\mathrm{cp}} = L$ samples, and also $\mathsf{n_T}L$ orthogonal pilots are inserted into every block. Assuming a stationary user to start having mobility, the channel coherence time becomes variable. In order to design the blocks such that they experience a nearly block-fading—and thus, circular channel—condition, bock length N_d for a flexible waveform is set to be 2% of the channel coherence time T_c.

Fig. 4 compares the transmission efficiency of the UW vs. CP based systems. One can see that in high mobility conditions where T_c is small, the transmission block in a CP based system, suffers from significant pilot overhead, e.g. in $T_c = 2$ms only 40% of the block is dedicated to useful data transmission, whereas in a UW system the efficiency is around 0.57 and 0.61 for UW based systems of $B = 10$ and $B = 50$ blocks, respectively. We also note that at stationary conditions where the block length can be set to larger values (and thus, longer duration), the efficiency curves tend to merge, although, the UW based system still has slightly higher efficiency due to its larger resource size S_{UW}.

Fig. 5 compares the pilot overhead (i.e. $1 - \eta$) of UW based system vs. CP based system of three radix-2 data FFT sizes N_d in percentage. We note that in the given scenario, $N_d = 128$ corresponds to current LTE symbol design with comb-type pilot pattern. As can be seen in Fig. 5, the overhead for $N_d = 128$ can be reduced from 37.3% of a CP based system to 26.8% of UW based system with $B = 50$ blocks. In addition, we also note that a UW based system can take advantage of per-block-synchronization in high mobility conditions, whereas in a CP based system, if a preamble based synchronization fails, $B_{\mathrm{CP}} = 10$ blocks would have been dropped.

B. Channel Estimation

Consider a 4×4 MIMO system in a scenario where the multi-path wireless channels are characterized by Rayleigh distribution. The PDP is exponentially decaying from 0 to -20 dB with the longest delay tap of 4.7μs (i.e. $L = 9$) and it is normalized to unity. The sampling rate is set to $F_s = 1.92$MHz. The UW length is set to $N_{\mathrm{u}} = 49$ samples, whereas the payload slots have a length of $N_d = 256$ samples. Thus, a *Payload-UW* block contains $N = 305$ samples yielding to a block rate of $F_b = 6.3$kHz. For the UW based transmission, we consider two cases of time-variant channels with maximum Doppler shift of $f_d = 300$Hz (i.e. 4.77% Normalized Doppler frequency) modeled according to [12], and a block fading case where $f_d \approx 0$, for the sake of optimality comparison. In UW based channel estimation, we use the last $N_{\mathrm{u}} - L$ IBI-free samples of the UW sequence, whereas in a CP-OFDM system, comb-type $N_{\mathrm{u}} - L$ pilots (with identical frequency spacing) in form of PN sequence that are mapped into QPSK symbols have been inserted into every OFDM block of length $N_{\mathrm{ofdm}} = 256$ samples. The MSE is calculated by comparing the estimation with respect to the channel realization at the center of the payload block.

As can be seen in Fig. 6, CP-OFDM has more than one order of magnitude larger error floor (i.e. due to the channel variations) compared to a UW based channel estimation. We justify this performance gap by considering the fact that the energy of the UW sequences are concentrated into $(N_{\mathrm{u}} - L)$-length time slot (i.e. 3.5% of the coherence time), whilst in CP-OFDM, the energy of the pilots are distributed over $N_{\mathrm{ofdm}} = 256$ samples (i.e. 22.3% of the coherence time). On the other hand, the UW based channel estimation follows almost the same performance of its time-invariant bound until SNR values of up to 30 dB. We also note that the theoretical MSE calculations via (10) coincides with "UW, $f_d = 0$" simulation, because it does not consider the error floor due to the channel variations.

V. Conclusions

In this paper, we have extended the conventional SISO UW frame design to MIMO applications, by considering a highly frequency-selective and time-varying channel. We have showed that a MIMO UW based system would have higher transmission efficiency and lower pilot overhead, when it is compared with a CP based system. Evaluating the channel estimation performances in frequency selective and time-varying conditions, we showed that a UW based channel estimation outperforms the conventional pilot-aided CP-OFDM channel estimation by one order of magnitude smaller MSE floor.

Acknowledgment

The computations were performed at the Center for Information Services and High Performance Computing (ZIH) of TU Dresden. This work has received funding from the European Union's Horizon 2020 research and innovation program under grant agreements no. 777137 (5GRANGE project) and no. 732174 (Orca project). Authors also acknowledge the Paris Seine Initiative (University of Cergy-Pontoise, CNRS, ENSEA, ETIS lab) for the support of the project through the ASIA Chair of Excellence Grant (PIA/ANR-16-IDEX-0008).

References

[1] L. Deneire, B. Gyselinckx, and M. Engels, "Training sequence vs. cyclic prefix a new look on single carrier communication," in *Globecom'00-IEEE. Global Telecommunications Conference. Conference Record (Cat. No. 00CH37137)*, vol. 2. IEEE, 2000, pp. 1056–1060.

[2] H. Witschnig, T. Mayer, A. Springer, A. Koppler, L. Maurer, M. Huemer, and R. Weigel, "A different look on cyclic prefix for SC/FDE," in *The 13th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, vol. 2. IEEE, 2002, pp. 824–828.

[3] J. Coon, M. Sandell, M. Beach, and J. McGeehan, "Channel and Noise Variance Estimation and Tracking Algorithms for Unique-Word based Single-Carrier Systems," *IEEE Transactions on Wireless Communications*, vol. 5, no. 6, pp. 1488–1496, 2006.

[4] M. Huemer, H. Witschnig, and J. Hausner, "Unique Word Based Phase Tracking Algorithms for SC/FDE-Systems," in *GLOBECOM'03. IEEE Global Telecommunications Conference (IEEE Cat. No. 03CH37489)*, vol. 1. IEEE, 2003, pp. 70–74.

[5] A. N. Mody and G. L. Stuber, "Synchronization for MIMO OFDM systems," in *IEEE Global Telecommunications Conference (GLOBECOM'01)*, vol. 1. IEEE, 2001, pp. 509–513.

[6] S. Ehsanfar, M. Matthé, D. Zhang, and G. Fettweis, "Interference-Free Pilots Insertion for MIMO-GFDM Channel Estimation," in *2017 IEEE Wireless Communications and Networking Conference (WCNC)*, March 2017, pp. 1–6.

[7] N. Suehiro and M. Hatori, "Modulatable orthogonal sequences and their application to SSMA systems," *IEEE Trans. Inf. Theory*, vol. 34, no. 1, pp. 93–100, 1988.

[8] T.-D. Chiueh and P.-Y. Tsai, *OFDM baseband receiver design for wireless communications*. John Wiley & Sons, 2008.

[9] X. Liu, H.-H. Chen, S. Chen, and W. Meng, "Symbol Cyclic-Shift Equalization Algorithm—A CP-Free OFDM/OFDMA System Design," *IEEE Transactions on Vehicular Technology*, vol. 66, no. 1, pp. 282–294, 2016.

[10] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Upper Saddle River, NJ: Prentice Hall, 1993, vol. 1.

[11] S. Ehsanfar, M. Chafii, and G. Fettweis, "Time-Variant Pilot- and CP-Aided Channel Estimation for GFDM," in *Proc. of the IEEE International Conference on Commun. (ICC'19)*. IEEE, 2019.

[12] R. H. Clarke, "A statistical theory of mobile-radio reception," *The Bell System Technical Journal*, vol. 47, no. 6, pp. 957–1000, July 1968.